CO3

a COnverter for proving COnfluence of COnditional term rewriting systems

Ver. 1.1

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Target and Main Function

Convert a normal 1-CTRS to a TRS by using

- the simultaneous unraveling $\mathbb{U}$

  [Marchiori, 96][Ohlebusch, 02][Gmeiner et al, 13]

- the SR transformation $\mathbb{SR}$ [Șerbănuță & Roșu, 06]
  - if the input CTRS is constructor-based, then the special bracket symbol and its rewrite rules are not introduced, i.e., the result is the same as by [Antoy et al, 03]

Theorem (theoretical background)

$\mathcal{R}$ is confluent if $\mathcal{R}$ is weakly left-linear (WLL) and

- *$\mathbb{U}(\mathcal{R})$ is confluent,* *[Gmeiner et al, 13]*

  or
- *$\mathbb{SR}(\mathcal{R})$ is confluent* *[Nishida et al, today (before lunch)]*

Example

$$\mathcal{R} = \left\{ \begin{array}{c} \mathsf{even}(0) \to \mathsf{true} \\ \mathsf{even}(\mathsf{s}(x)) \to \mathsf{true} \Leftarrow \mathsf{even}(x) \twoheadrightarrow \mathsf{false} \\ \mathsf{even}(\mathsf{s}(x)) \to \mathsf{false} \Leftarrow \mathsf{even}(x) \twoheadrightarrow \mathsf{true} \end{array} \right\}$$

$$\mathbb{U}(\mathcal{R}) = \left\{ \begin{array}{c} \mathsf{even}(0) \to \mathsf{true} \\ \mathsf{even}(\mathsf{s}(x)) \to U_1(\mathsf{even}(x), x) \\ U_1(\mathsf{false}, x) \to \mathsf{true} \\ U_1(\mathsf{true}, x) \to \mathsf{false} \end{array} \right\}$$

$$\mathbb{SR}(\mathcal{R}) = \left\{ \begin{array}{c} \overline{\mathsf{even}}(0, z) \to \mathsf{true} \\ \overline{\mathsf{even}}(\mathsf{s}(x), \bot) \to \overline{\mathsf{even}}(\mathsf{s}(x), \mathsf{even}(x)) \\ \overline{\mathsf{even}}(\mathsf{s}(x), \mathsf{false}) \to \mathsf{true} \\ \overline{\mathsf{even}}(\mathsf{s}(x), \mathsf{true}) \to \mathsf{false} \end{array} \right\}$$

- $\mathbb{U}(\mathcal{R})$ and $\mathbb{SR}(\mathcal{R})$ are orthogonal, and thus, $\mathcal{R}$ is confluent

How to Prove/Disprove Confluence of $\mathcal{R}$

Implemented Criteria for Confluence

- Orthogonality of $\mathbb{U}(\mathcal{R})$ or $\mathbb{SR}(\mathcal{R})$ if $\mathcal{R}$ is WLL
- Termination and CP-joinability of $\mathbb{U}(\mathcal{R})$ or $\mathbb{SR}(\mathcal{R})$ if $\mathcal{R}$ is WLL
  - the emptiness of the union of the SCCs in EDG
  - the simplest reduction pair
    - $s \geq t$ if $|s| \geq |t|$ and $\forall x \in \mathcal{V}.\ |s|_x \geq |t|_x$
    - $s > t$ if $|s| > |t|$ and $\forall x \in \mathcal{V}.\ |s|_x \geq |t|_x$

Implemented Criterion for Non-confluence

Existence of unconditional CP (s, t) such that

- s and t are ground irreducible on $\mathcal{R}_u$ $(= \{l \to r \mid l \to r \Leftarrow c \in \mathcal{R}\})$, or
- $CAP(s)$ and $CAP(t)$ is not unifiable

Remark

- The feature of CO3 is syntactic analysis
  - the power of proving termination and confluence of TRSs is weak
  - CO3 will rely on other tools
- CO3 website:

  `http://www.trs.cm.is.nagoya-u.ac.jp/co3/`

  - Ver. 1.0 is now available
  - Updated to Ver. 1.1 soon