Nrbox

System Description for CoCo 2016

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nrbox: **n**ominal **r**ewriting tool**box**

Confluence Tool for "*Nominal Rewriting Systems*"

- Written in SML/NJ, about 4500 loc
- `http://www.nue.ie.niigata-u.ac.jp/tools/nrbox/`
- Requires an external *termination prover* for first-order TRSs.
- Entrant for Demo(NRS) category.

Nominal Rewriting (Fernández & Gabbay, 2007)

- Extension of first-order term rewriting
- Binding mechanism
  - Nominal approach (Gabbay & Pitts, 2002)
- α-equivalence is dealt with at object-level
  - In contrast to traditional higher-order rewriting, which uses λ-calculus as meta-calculus.

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Example.

$$\begin{array}{rcll} (\forall a.P) \wedge Q & \equiv & \forall a.(P \wedge Q) & (a \notin FV(Q)) \\ P \wedge (\forall a.Q) & \equiv & \forall a.(P \wedge Q) & (a \notin FV(P)) \end{array}$$

Nominal rewriting system (NRS for short)

$$\left\{ \begin{array}{lcll} \mathsf{a}\#Q & \vdash & \mathtt{and}(\mathtt{forall}([\mathsf{a}]P), Q) & \to & \mathtt{forall}([\mathsf{a}]\mathtt{and}(P, Q)) \\ \mathsf{a}\#P & \vdash & \mathtt{and}(P, \mathtt{forall}([\mathsf{a}]Q)) & \to & \mathtt{forall}([\mathsf{a}]\mathtt{and}(P, Q)) \end{array} \right.$$

Rewriting by NRSs

Definition [rewrite relation]

$\Delta \vdash s \to_{\langle R,\pi,p,\sigma\rangle} t \stackrel{\text{def}}{\Longleftrightarrow} \Delta \vdash \nabla^{\pi}\sigma,\ \Delta \vdash s|_p \approx_\alpha l^{\pi}\sigma,\ t = s[r^{\pi}\sigma]_p.$

$\Delta \vdash s \to_{\mathcal{R}} t \stackrel{\text{def}}{\Longleftrightarrow} \Delta \vdash s \to_{\langle R,\pi,p,\sigma\rangle} t$ for some $R \in \mathcal{R}, \pi, p, \sigma$.

$$\begin{array}{ll}
 & \mathtt{and}(\mathtt{forall}([\mathsf{a}]p(\mathsf{a})), \mathtt{forall}([\mathsf{a}]q(\mathsf{a}))) \\
\to_{R_1} & \mathtt{forall}([\mathsf{a}]\mathtt{and}(p(\mathsf{a}), \mathtt{forall}([\mathsf{a}]q(\mathsf{a})))) \\
\approx_\alpha & \mathtt{forall}([\mathsf{a}]\mathtt{and}(p(\mathsf{a}), \mathtt{forall}([\mathsf{b}]q(\mathsf{b})))) \\
\to_{R_2^{(\mathsf{a}\ \mathsf{b})}} & \mathtt{forall}([\mathsf{a}]\mathtt{forall}([\mathsf{b}]\mathtt{and}(p(\mathsf{a}), q(\mathsf{b}))))
\end{array}$$

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$$\begin{array}{ll} & \mathtt{and}(\mathtt{forall}([\mathsf{a}]p(\mathsf{a})), \mathtt{forall}([\mathsf{a}]q(\mathsf{a}))) \\ \rightarrow_{R_1} & \mathtt{forall}([\mathsf{a}]\mathtt{and}(p(\mathsf{a}), \mathtt{forall}([\mathsf{a}]q(\mathsf{a})))) \\ \approx_\alpha & \mathtt{forall}([\mathsf{a}]\mathtt{and}(p(\mathsf{a}), \mathtt{forall}([\mathsf{b}]q(\mathsf{b})))) \\ \rightarrow_{R_2^{(\mathsf{a}\ \mathsf{b})}} & \mathtt{forall}([\mathsf{a}]\mathtt{forall}([\mathsf{b}]\mathtt{and}(p(\mathsf{a}), q(\mathsf{b})))) \end{array}$$

Definition [Church-Rosser modulo $\approx_\alpha$ (CRM for short)]

$$t_1 \quad (\leftarrow \cup \rightarrow \cup \approx_\alpha)^* \quad t_2$$

$$t_1 \xrightarrow{*} s_1 \approx_\alpha s_2 \xleftarrow{*} t_2$$

Implemented Confluence Criteria

Proposition [SKAT, RTA 2015]

Abstract skeleton preserving orthogonal NRSs are CRM.

Proposition [SKAT, SCSS 2016]

Linear uniform NRSs are CRM if $\Gamma \vdash u \rightarrow^{=} \circ \approx_\alpha \circ \leftarrow^{*} v$ and $\Gamma \vdash u \rightarrow^{*} \circ \approx_\alpha \circ \leftarrow^{=} v$ for any BCP $\Gamma \vdash \langle u, v \rangle$.

Proposition [SKAT, SCSS 2016]

Terminating uniform NRSs are CRM if and only if $\Gamma \vdash u\ (\rightarrow^{*} \circ \approx_\alpha \circ \leftarrow^{*})\ v$ for any BCP $\Gamma \vdash \langle u, v \rangle$.

Proposition [KAT, 2016]

Left-linear uniform NRSs are CRM if $\Gamma \vdash u \twoheadrightarrow\!\!\!\!\!+ \circ \approx_\alpha v$ $(u \twoheadrightarrow\!\!\!\!\!+ \circ \approx_\alpha \circ \leftarrow^{*} v)$ for any inner (resp. outer) BCP $\Gamma \vdash \langle u, v \rangle$.